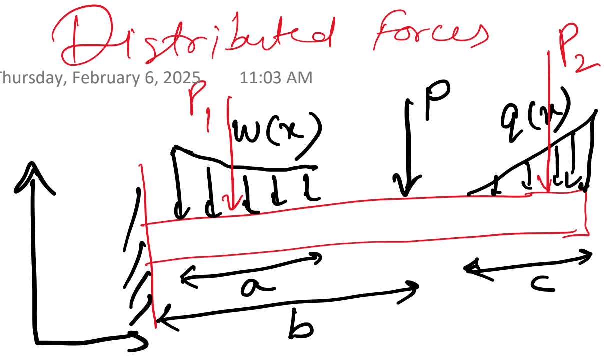
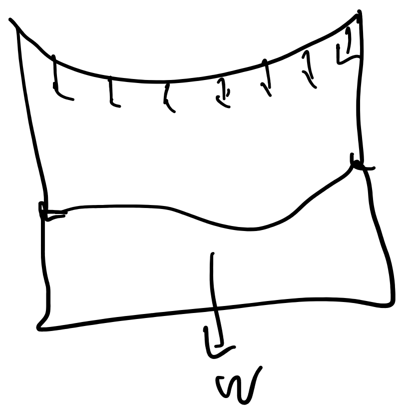
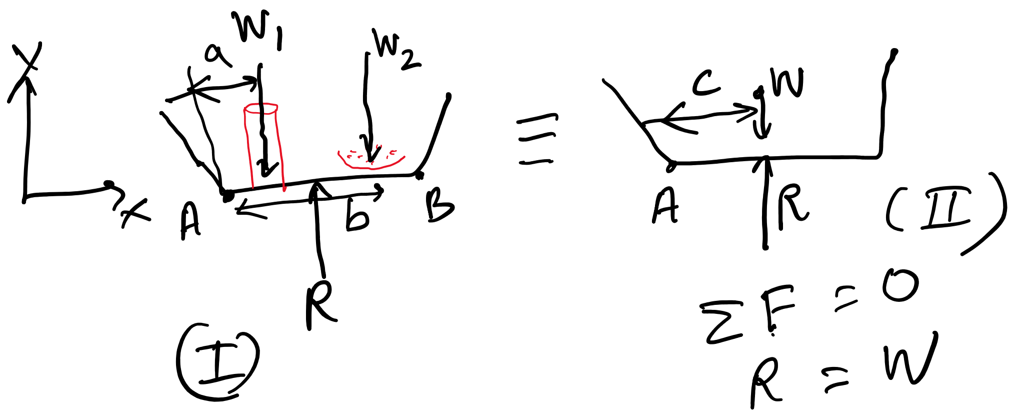
Distributed forces



$$\sum \vec{F} = 0$$
$$\sum \vec{M} = 0$$

Center of distribution





$$\sum F = 0$$
$$R = W$$

$$\left(\sum \vec{F}\right)_I = \left(\sum \vec{F}\right)_{II}$$

$$W_1 + W_2 = W$$

$$\left(\sum M_A\right)_I = \left(\sum M_A\right)_{II}$$

$$W_1 a + W_2 b = W \times c$$

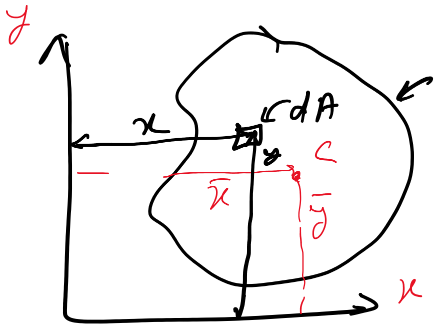
$$c = \frac{W_1 a + W_2 b}{W}$$

$$= \frac{W_1 a + W_2 b}{W_1 + W_2}$$

$$\bar{x} = \frac{\sum W_i a_i}{\sum W_i}$$

$$\bar{x} = \frac{\sum W_i x_i}{\sum W_i}$$

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

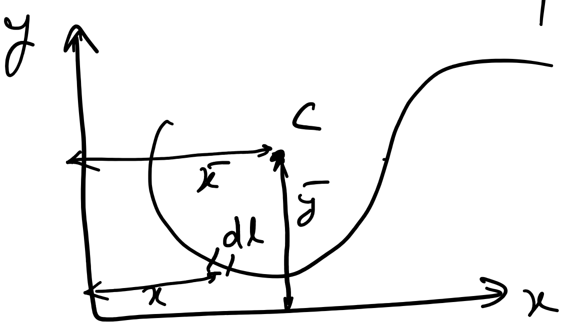


$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\underbrace{\sum A_i}_{A}}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\underbrace{\sum A_i}_{A}}$$

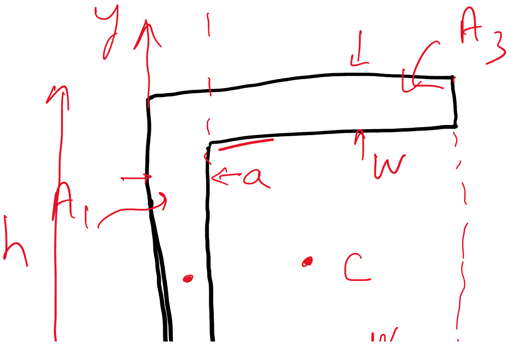
$$A\bar{x} = \sum_{i=1}^{n} A_i x_i$$

$$\bar{x} = \frac{\int dA\, x}{\int dA} \qquad \bar{y} = \frac{\int dA\, \bar{y}}{\int dA}$$



$$\bar{x} = \frac{\int dl\, x}{\int dl}$$

$$\bar{y} = \frac{\int dl\, y}{\int dl}$$

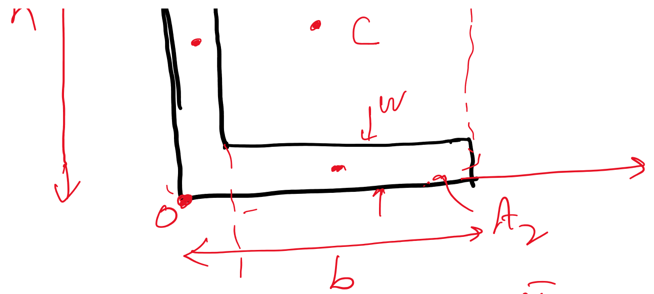


$A_1 = h a$

$A_2 = w(b-a)$

$A_3 = w(b-a)$

$\bar{x}_1 = \frac{a}{2}, \quad \bar{y}_1 = \frac{h}{2}$

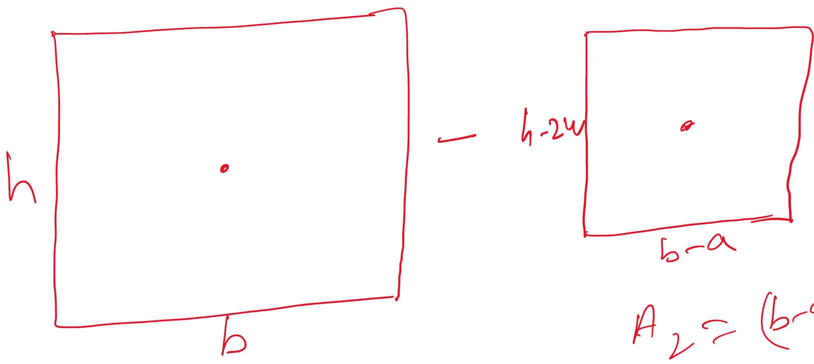


$\bar{x}_1 = \frac{a}{2}$, $\bar{y}_1 = \frac{h}{2}$

$\bar{x}_2 = \frac{b+a}{2}$, $\bar{y}_2 = \frac{w}{2}$

$\bar{x}_3 = \frac{b+a}{2}$; $\bar{y}_3 = h - \frac{w}{2}$

$$\bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i} \; ; \; \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

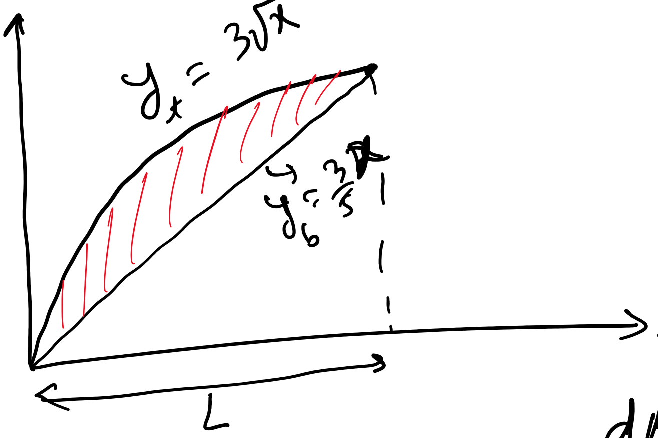


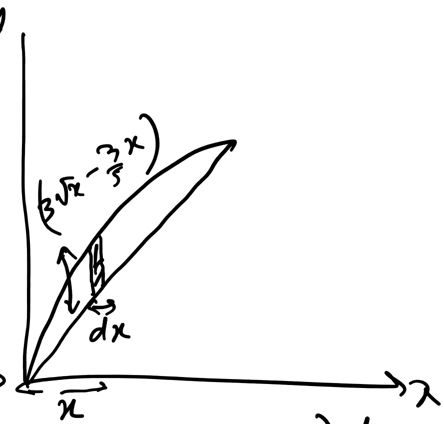
$A_1 = bh$

$A_2 = (b-a)(h-2w)$

$\bar{x} =$

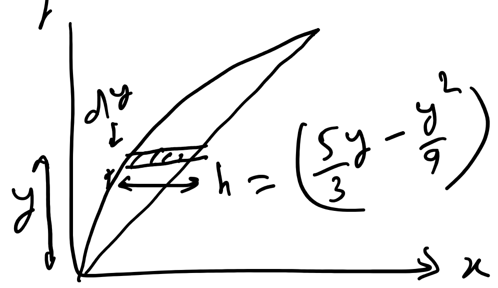
$\bar{y} =$





$$dA = \left(3\sqrt{x} - \frac{3}{5}x\right)dx$$

$$\bar{x} = \frac{\int_0^L dA\, x}{\int_0^L dA} \; ; \; \bar{y} = \frac{\int_0^L dA\, y}{\int_0^L dA}$$

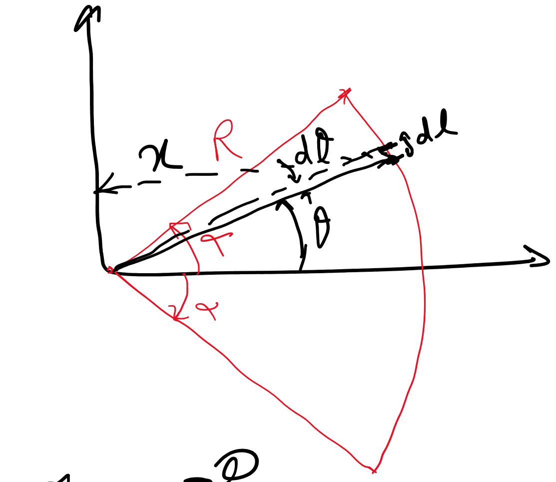


$$dA = \left(\frac{5}{3}y - \frac{y^2}{9}\right)dy$$

$$dA = \left(\frac{2}{3} \cdots \frac{\cdots}{9}\right) d\theta$$

$$\bar{y} = \frac{\int dA\, y}{\int dA}$$

Centroid of line



$$dl = R\,d\theta$$

$$\bar{x} = \int dl\, x \Big/ \int dl$$

$$x = R\cos\theta$$

$$\bar{x} = \frac{\int_{-\alpha}^{\alpha} (R\,d\theta)(R\cos\theta)}{\int_{-\alpha}^{\alpha} R\,d\theta}$$

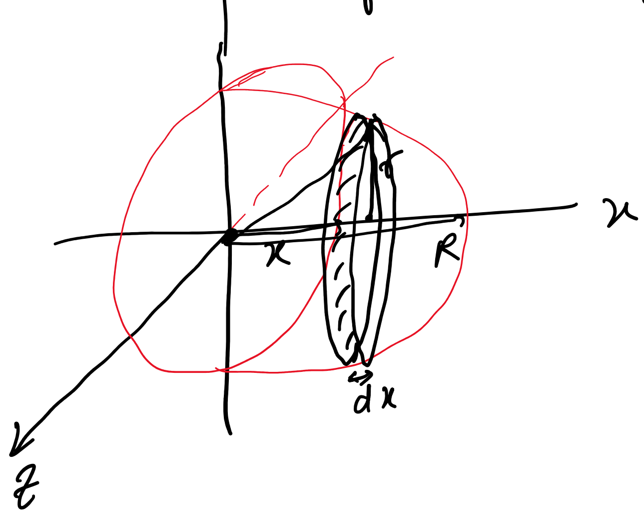
$$= \frac{\int_{-\alpha}^{\alpha} R^2\cos\theta\, d\theta}{\int_{-\alpha}^{\alpha} R\,d\theta}$$

$$\bar{y} = \frac{\int_{-\alpha}^{\alpha} R^2 \sin\theta\, d\theta}{\int_{-\alpha}^{\alpha} R\,d\theta} = 0$$

$$\boxed{\bar{x} = \frac{R\sin\alpha}{\alpha}}$$

Centroid of a Volume

$$r^2 = R^2 - x^2$$



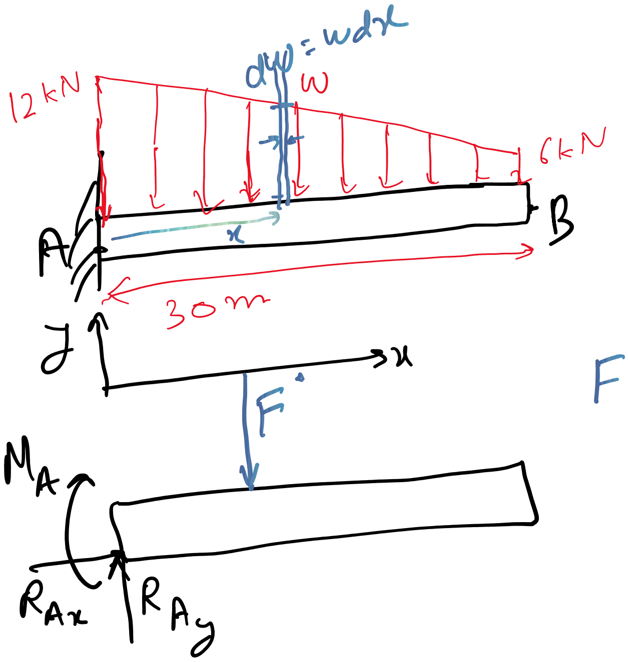
$$\bar{x} = \frac{\int (dV)\, x}{\int dV}$$

$$= \frac{\int_0^R (\pi r^2 dx)\, x}{\int_0^R \pi r^2 dx}$$

$$= \frac{\int_0^R \pi (R^2 - x^2)\, x\, dx}{\int_0^R \cdots (R^2 - x^2)\, dx}$$

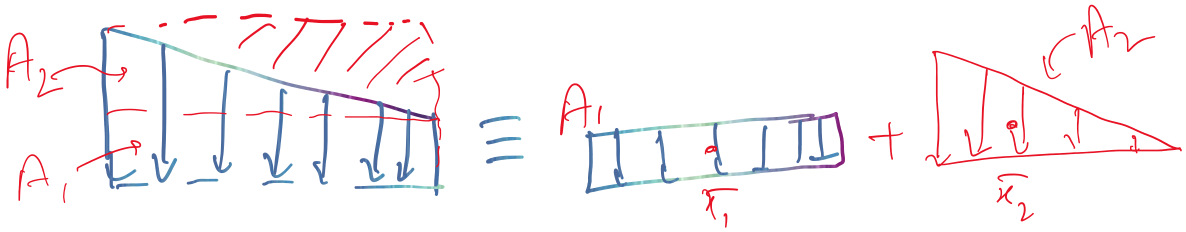
z

$$\frac{}{\int_0^R \pi (R^2 - x^2)\, dx}$$

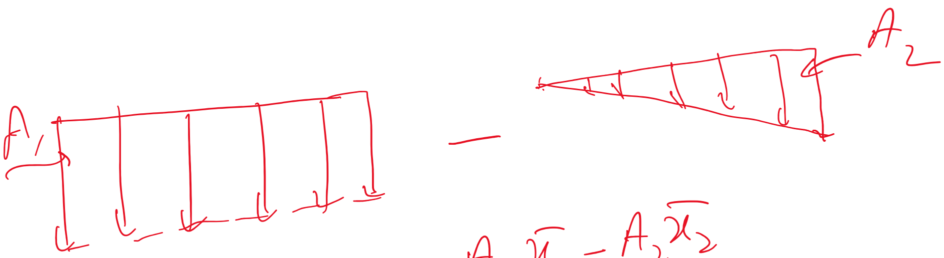


$$w(x) = (12 - 0.2x)\ \text{kN/m}$$

$$F = \int_0^L w\, dx = \int_0^L dW$$



$$\bar{x} = \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2}{A_1 + A_2}$$



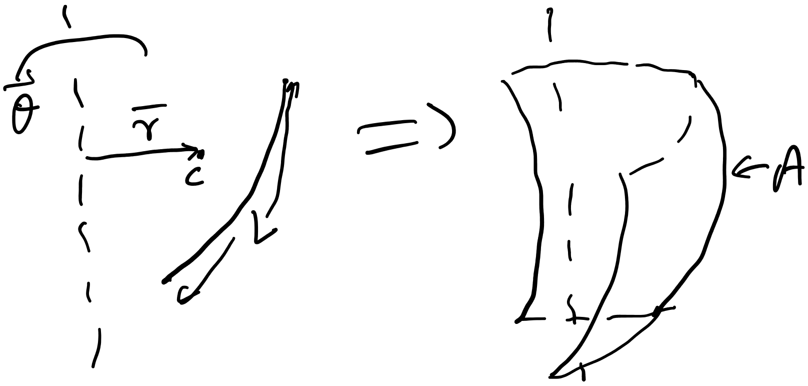
$$\bar{x} = \frac{A_1 \bar{x}_1 - A_2 \bar{x}_2}{A_1 - A_2}$$

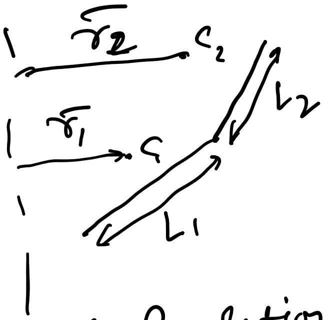
Theorem of Pappus & Guldinus

Surface of Revolution

$$A = L \bar{r} \theta$$

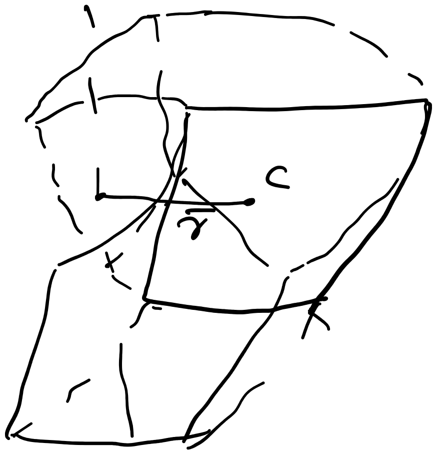
$A = L\bar{r}\theta$





$\Rightarrow A = \theta \sum_i \bar{r}_i L_i$

Volume of Revolution

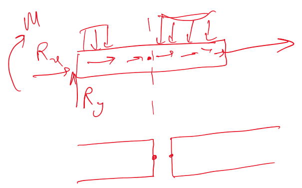


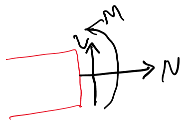
$V = \theta A \bar{r}$

$V = \theta \sum_i A_i \bar{r}_i$

Internal Forces

Monday, February 10, 2025 10:51 AM

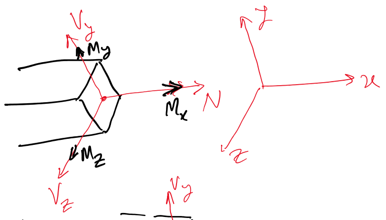


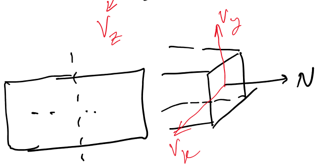


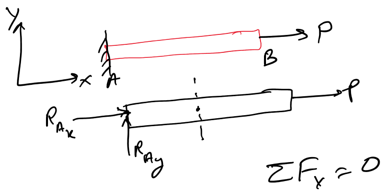
$N =$ axial Force
Normal Force

$V =$ Shear Force

$M =$ Bending Moment





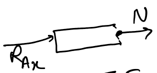


$\Sigma F_y = 0$

$\Rightarrow R_{Ay} = 0$

$\Sigma F_x = 0$

$R_{Ax} = -P$



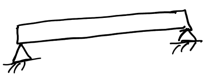
$\Sigma F_x = 0$

$N = -R_{Ax} = P$

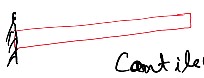
$\Sigma F_x = 0$

$N = P$

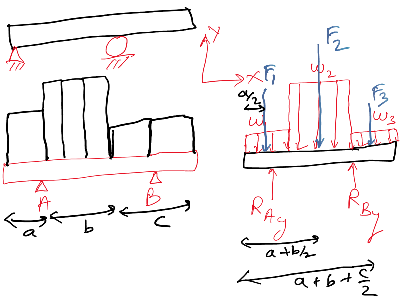
Beams



Simply Supported Beam



Cantilever

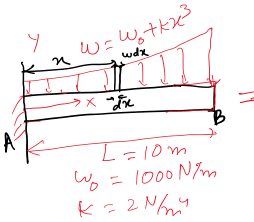


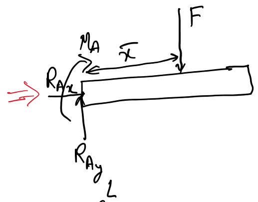
$-w_0 + kx^3$

F

$\overline{a+b/2}$

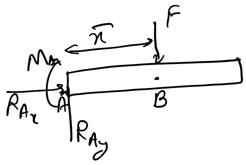
$\overleftarrow{a+b+\frac{c}{2}}$





$$F = \int_0^L w\,dx = \int_0^L (w_0 + kx^3)\,dx = w_0 L + \frac{k}{4} L^4$$

$$\bar{x} = \frac{\int_0^L (w\,dx)\,x}{F} =$$



$$\Sigma M_A = 0$$

$$M_A + F\bar{x} = 0$$

$$M_A = -F\bar{x}$$

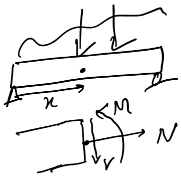
$$\Sigma F_x = 0 \Rightarrow R_{A_x} = 0$$

$$\Sigma F_y = 0 \Rightarrow R_{A_y} = F$$

$$\Sigma M_B = 0$$

$$M_A + R_{A_y} \times \bar{x} = 0$$

$$M_A = -R_{A_y}\bar{x} = -F\bar{x}$$



$M, N, V \Rightarrow$ function of x

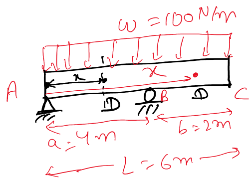
$\Downarrow$

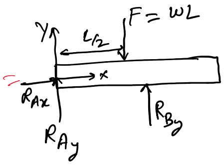
Moment & Shear Force

$\Downarrow$ Diagram

Bending Moment Diagram (BMD)

Shear Force Diagram (SFD)





$M(x) = ?$

$V(x) = ?$

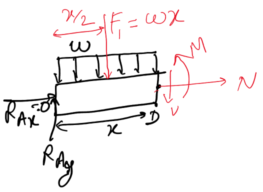
$$\Sigma M_A = 0$$

$$\Sigma F_x = 0 \Rightarrow R_{A_x} = 0$$

$$\Sigma F_y = 0$$

$$R_{Ay} = \frac{F}{2}\left(1 - \frac{b}{a}\right)$$

$$R_{By} = \frac{FL}{2a}$$



$$\Sigma F_x = 0 \Rightarrow N = 0$$

$$\Sigma M_D = 0$$

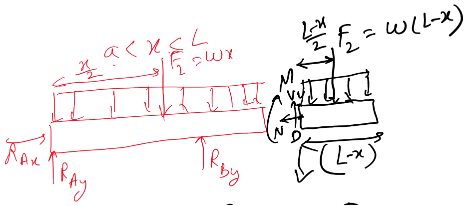
$$\Rightarrow R_{A_y} \times x - F_1 \frac{x}{2} - M = 0$$

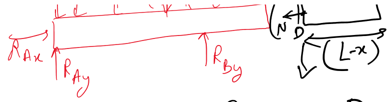
$$\Rightarrow M = \frac{F}{2}\left(1 - \frac{b}{a}\right)x - \frac{wx^2}{2}$$

$$\Sigma F_y = 0$$

$$\Rightarrow V = \frac{F}{2}\left(1 - \frac{b}{a}\right) - wx \Bigg\} \Rightarrow 0 \le x \le a$$

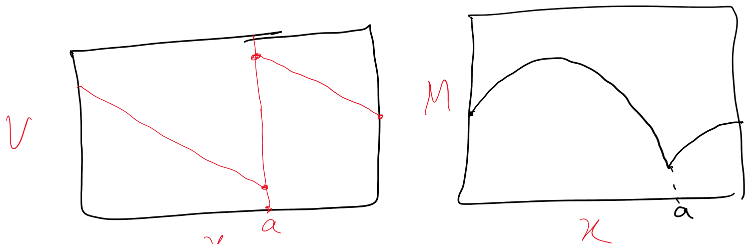
$a < x \le L$

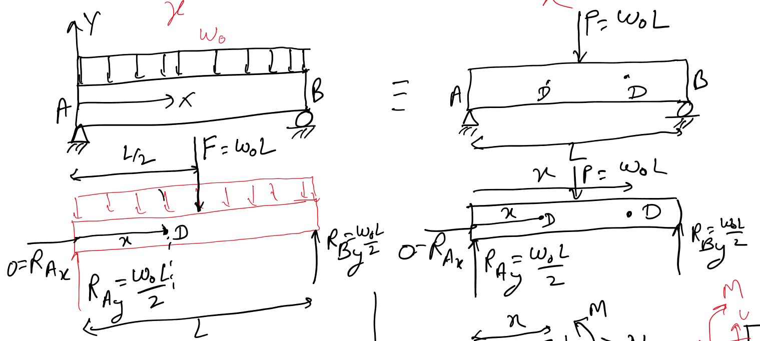


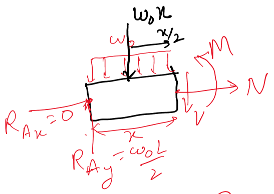


$$V = w(L-x)$$
$$M = -\frac{w}{2}(L-x)^2$$

$$\sum F_y = 0$$
$$\sum F_x = 0$$
$$\sum M_D = 0$$



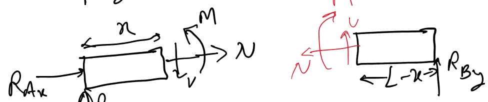




$$\sum F_x = 0$$
$$N = -R_{Ax} = 0$$

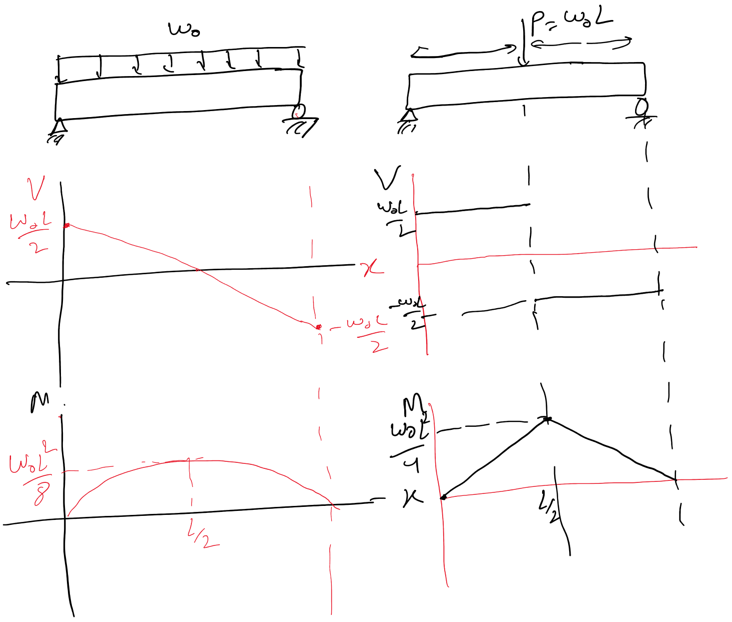
$$\sum F_y = 0$$
$$R_{Ay} - w_0 x - V = 0$$
$$V = \frac{w_0 L}{2} - w_0 x$$

$$\sum M_D = 0$$
$$R_{Ay} \times x - w_0 x \times \frac{x}{2} - M = 0$$
$$M = \frac{w_0 x}{2}(L-x)$$

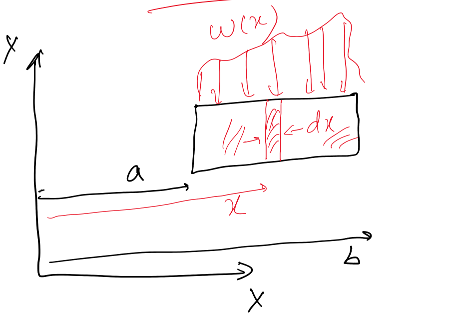


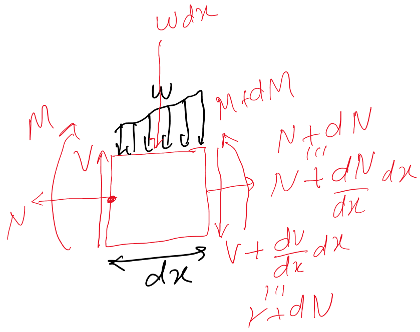
$$\sum M_D = 0$$
$$R_{Ay} x - M = 0$$
$$M = \frac{w_0 L}{2} x$$
$$\sum F_x = 0$$
$$N = 0$$
$$\sum F_y = 0$$
$$V = R_{Ay} = \frac{w_0 L}{2}$$

$$V = -\frac{w_0 L}{2}$$
$$N = 0$$
$$M = \frac{w_0 L}{2}(L-x)$$



General Loading





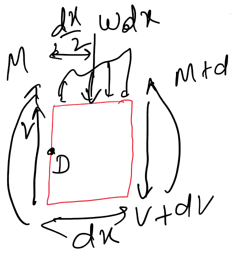
$$N+dN$$
$$dN = \frac{\partial N}{\partial x}\Delta x$$

$$\Sigma F_x = 0$$
$$N+dN-N = 0 \Rightarrow dN = 0$$

$$\Sigma F_y = 0$$
$$V - w\,dx - V - dV = 0$$
$$\Rightarrow dV = -w\,dx$$
$$\boxed{\frac{dV}{dx} = -w}$$

$$\int_{V_a}^{V} dV = \int_{x_a}^{x} -w\,dx$$
$$V - V_a = -\int_{x_a}^{x} w\,dx$$
$$\boxed{V = V_a - \int_{x_a}^{x} w\,dx}$$

$$V = V_0 - \int_{0}^{x} w_0\,dx$$
$$= \frac{w_0 L}{2} - w_0 x$$

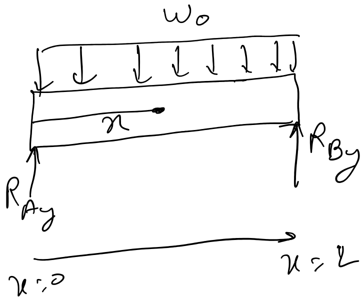


$$\Sigma M_D = 0$$
$$\cancel{M} + w\,dx \times \frac{dx}{2} - \cancel{M} - dM + (V+dV)\,dx = 0$$
$$-dM + V\,dx + \cancel{dV\,dx} = 0$$
$$dM = V\,dx$$
$$\frac{dM}{dx} = V$$
$$\boxed{\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w}$$

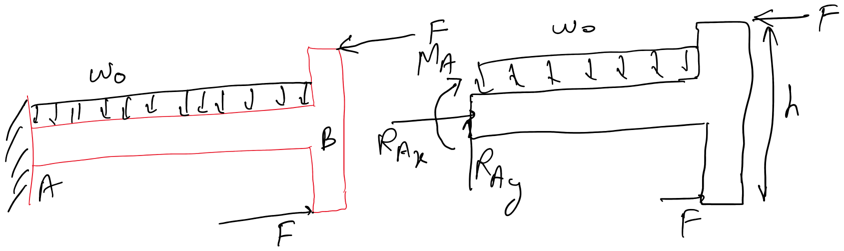
$\overline{d}x$

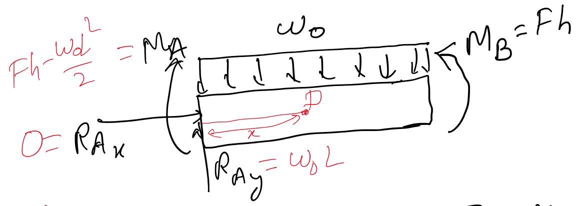
$$\boxed{\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w}$$

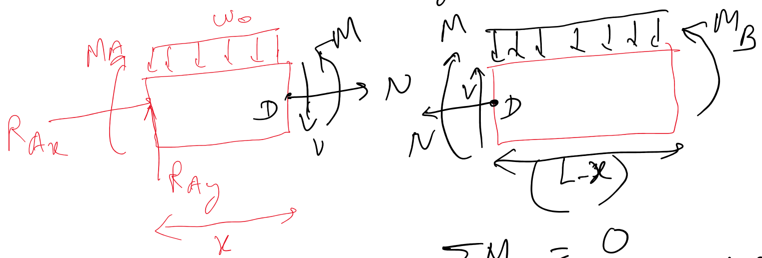
$$M = M_a + \int_{x_a}^{x} V\,dx$$



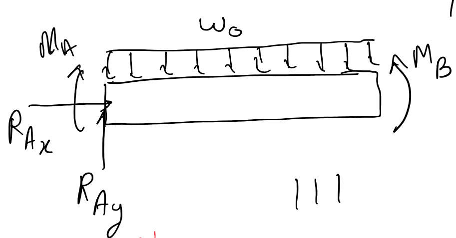
$$V = V_o - \int_0^x w_o\,dx$$
$$= \frac{w_oL}{2} - w_o x$$



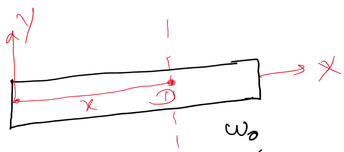


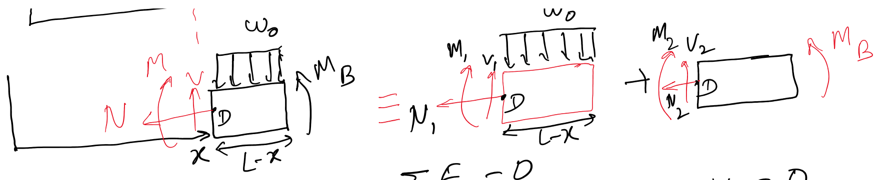


$$\Sigma M_D = 0$$
$$\Rightarrow M = Fh - \frac{w_o(L-x)^2}{2}$$
$$\Sigma F_y = 0$$
$$V = w_o(L-x)$$
$$N = 0$$



III





$$M = M_1 + M_2$$
$$V = V_1 + V_2$$
$$N = N_1 + N_2 = 0$$

$\Sigma F_x = 0$

$N_1 = 0$

$\Sigma F_y = 0$

$V_1 = w_0 (L-x)$

$\Sigma M_D = 0$

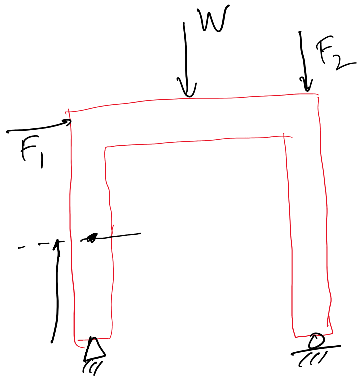
$M_1 = -\frac{w_0}{2}(L-x)^2$

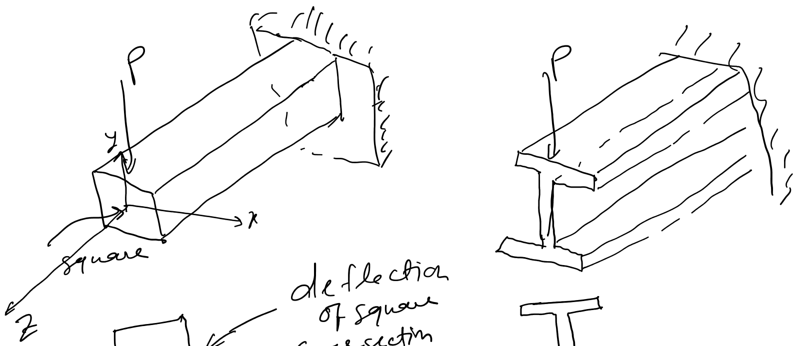
$N_2 = 0$

$V_2 = 0$

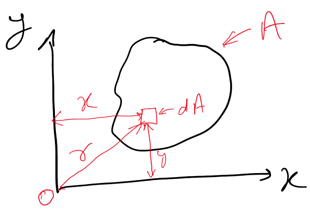
$M_2 = M_B$
$= Fh$

$$V = w_0 (L-x)$$
$$M = Fh - \frac{w_0}{2}(L-x)^2$$





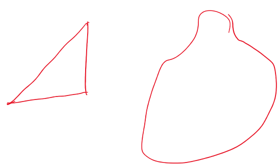
deflection of square cross-section is more than the I-section



$$I_x = \int_A y^2 \, dA$$
$$I_y = \int_A x^2 \, dA$$

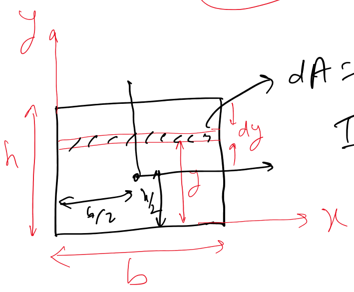
Polar moment of inertia

$$J_O = \int_A r^2 \, dA$$
$$= \int x^2 \, dA + \int y^2 \, dA$$
$$= I_x + I_y$$



$$I_{xy} = \int xy \, dA$$

y_a

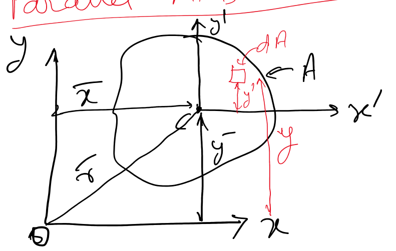


$dA = b\,dy$

$$I_x = \int y^2\,dA = \int_0^h b y^2\,dy = \frac{bh^3}{3}$$

$$I_y = \frac{hb^3}{3}$$

Parallel Axis theorem



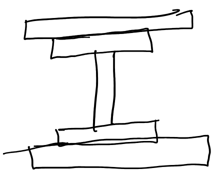
$$I_x = \int y^2\,dA = \int (y'+\bar{y})^2\,dA$$

$$= \underbrace{\int y'^2\,dA}_{I_{x'}} + \underbrace{\int \bar{y}^2\,dA}_{\bar{y}^2 A} + \underbrace{\int 2y'\bar{y}\,dA}_{2\bar{y}\cancel{\int y'\,dA}^{\,0}}$$

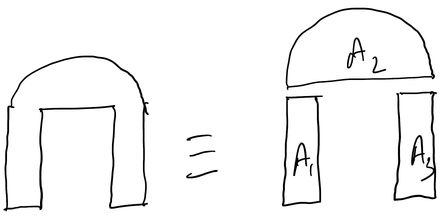
$$I_x = I_{x'} + A\bar{y}^2$$

$$I_y = I_{y'} + A\bar{x}^2$$

$$J_O = J_C + A\bar{r}^2$$

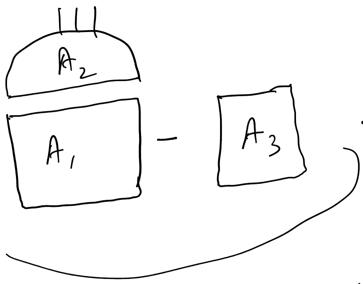


$$I_x = \sum_{i=1}^{N} \left(I_{x'_i} + A_i \bar{y}_i^2\right)$$



$A = A_1 + A_2 + A_3$

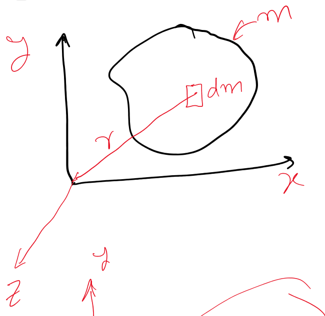
$$I_x = I_{x'_1} + I_{x'_2} + I_{x'_3} + A_1\bar{y}_1^2 + A_2\bar{y}_2^2 + A_3\bar{y}_3^2$$



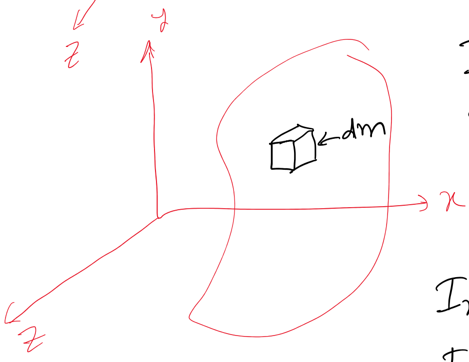
$\Rightarrow A = A_1 + A_2 - A_3$

Mass moment of inertia

Mass moment of inertia



$$I_z = \int r^2 \, dm$$
$$= \int (x^2 + y^2) \, dm$$



$$I_x = \int (y^2 + z^2) \, dm$$
$$I_y = \int (x^2 + z^2) \, dm$$
$$I_z = \int (x^2 + y^2) \, dm$$

$$I_{xy} = \int xy \, dm$$
$$I_{yz} = \int yz \, dm$$
$$I_{zx} = \int zx \, dm$$