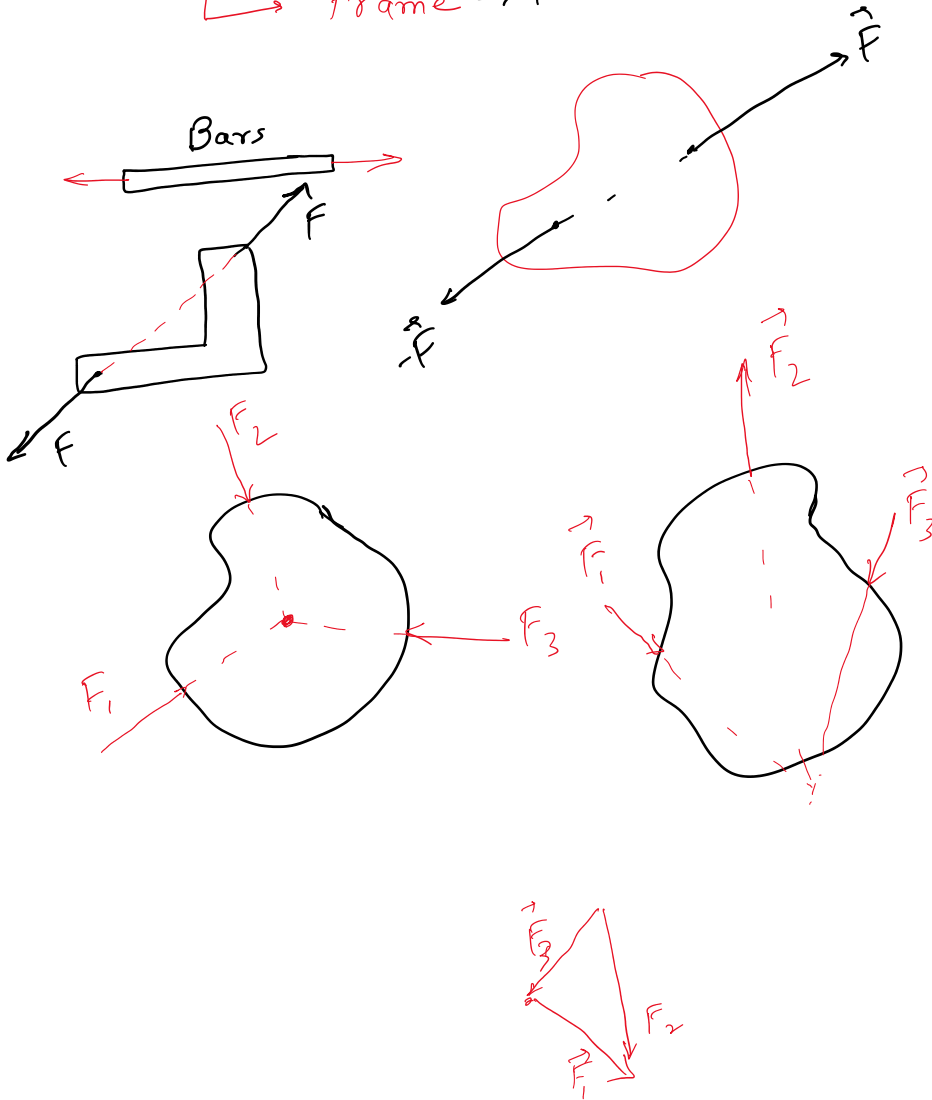


Structures

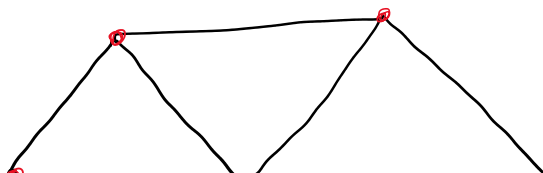
Wednesday, January 22, 2025 9:04 AM

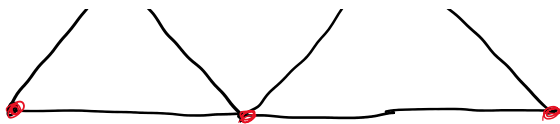
- Truss → Two force members
- Frame → Three force members



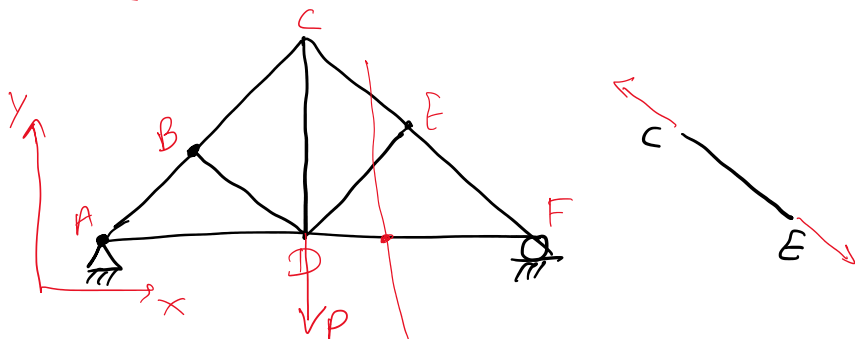
Truss

- Plane Truss
- Connected by frictionless pins
- Each member have no more than two joints
- Forces are applied only at joints
- Weight of individual member are negligible.



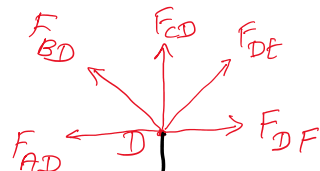


Analysis of Truss Structure



$$\sum F_x = 0$$

$$\sum F_y = 0$$

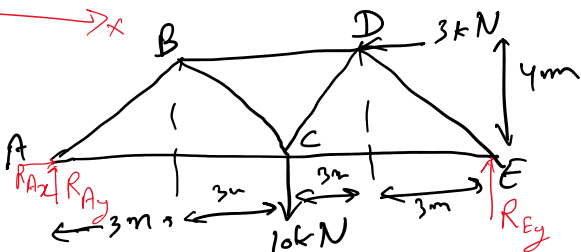
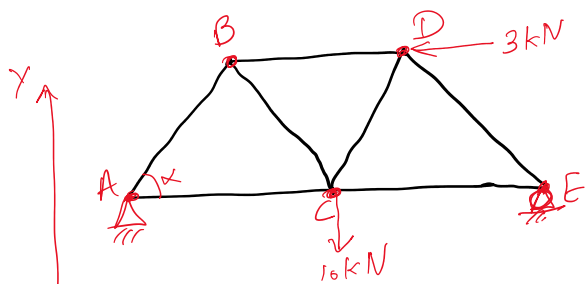


Method of Joints



Method of section

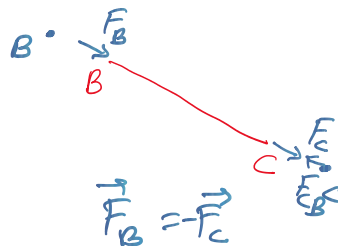
Method of Joints

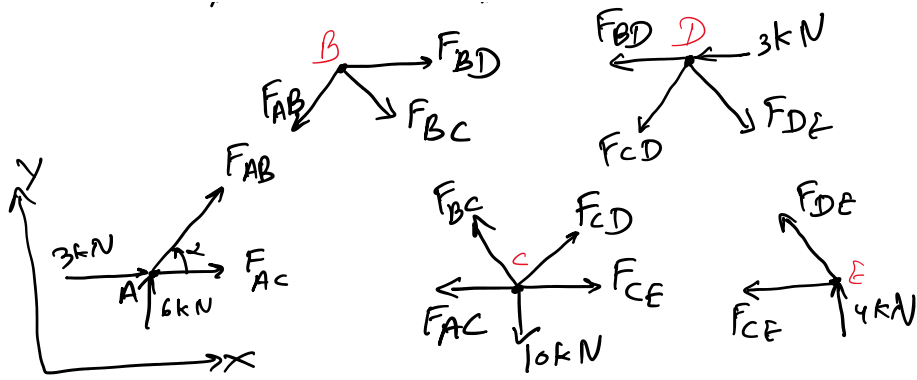


$$\sum M_A = 0 \Rightarrow R_{Ey} = 4 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow R_{Ay} = 6 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow R_{Ax} = 3 \text{ kN}$$






Joint A:


$$\begin{aligned} \sum F_x &= 0 & \sum F_y &= 0 \\ 3\text{kN} + F_{AB} \cos \alpha + F_{AC} &= 0 & F_{AB} \sin \alpha + 6\text{kN} &= 0 \\ F_{AC} &=? & \Rightarrow F_{AB} &=? \end{aligned}$$

Joint B:



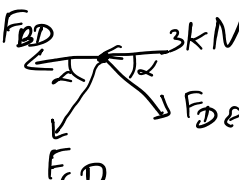
$$\begin{aligned} \sum F_x &= 0 & \sum F_y &= 0 \\ F_{BD} + F_{BC} \cos \alpha - F_{AB} \cos \alpha &= 0 & F_{AB} \sin \alpha + F_{BC} \sin \alpha &= 0 \\ F_{BD} &=? & F_{BC} &= -F_{AB} \end{aligned}$$

Joint E:

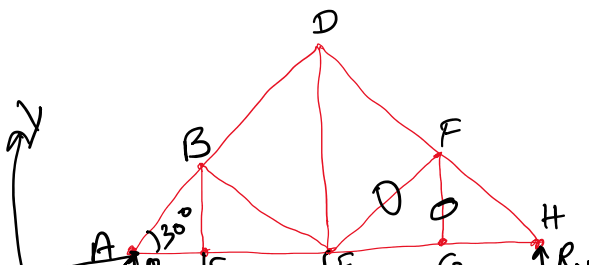


$$\begin{aligned} \sum F_x &= 0 & \sum F_y &= 0 \\ -F_{DE} \sin \alpha - F_{CE} &= 0 & F_{DE} \sin \alpha + 4\text{kN} &= 0 \\ F_{CE} &=? & F_{DE} &=? \end{aligned}$$

Joint D:



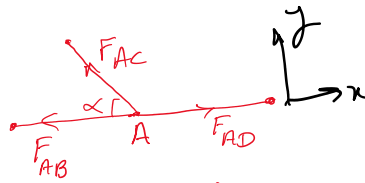
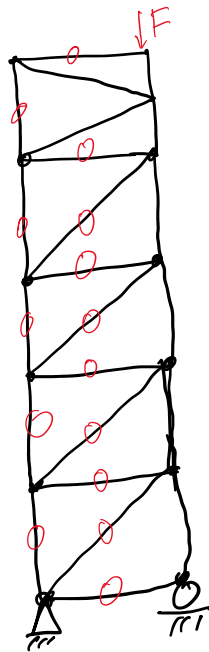
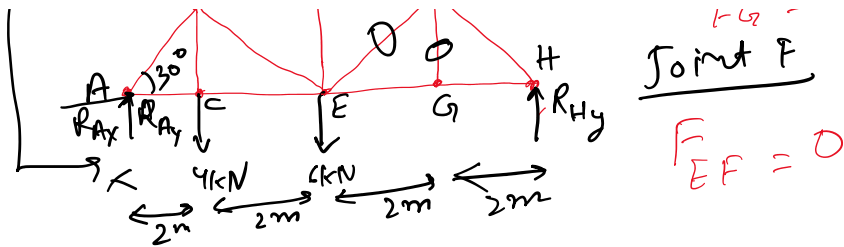
$$\begin{aligned} \sum F_y &= 0 \\ F_{CD} \sin \alpha + F_{DE} \sin \alpha &= 0 \\ F_{CD} &=? \end{aligned}$$



Joint G:

$$F_{FG} = 0$$

Joint F:



For joint A

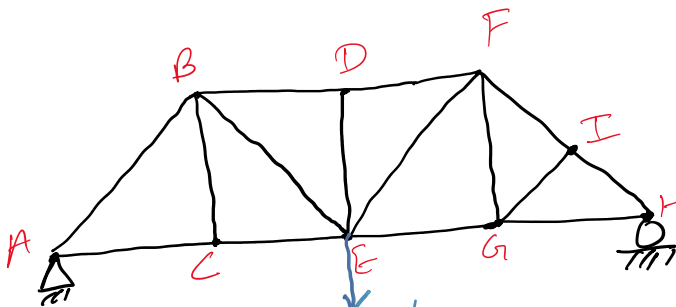
$$\sum F_y = 0$$

$$F_{AC} \sin \alpha = 0$$

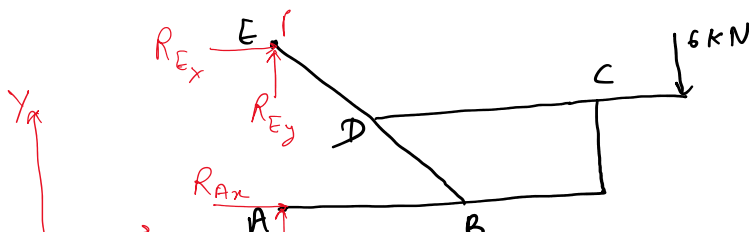
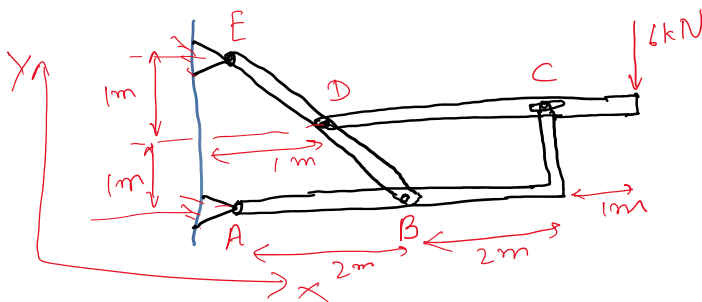
$$F_{AC} = 0$$

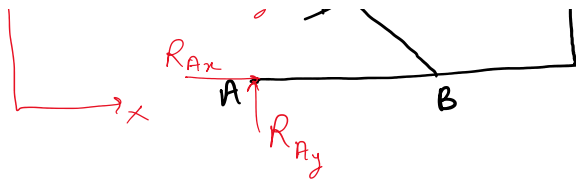
Sufficient Condition

- ↳ Has 3 members connected
- ↳ 2 are collinear
- ↳ No external force applied.



BC as zero force member
DE //





$$\sum M_E = 0$$

$$\Rightarrow R_{Ay} \times 2\text{m} - 6\text{kN} \times 5\text{m} = 0$$

$$\Rightarrow R_{Ay} = 15\text{kN}$$

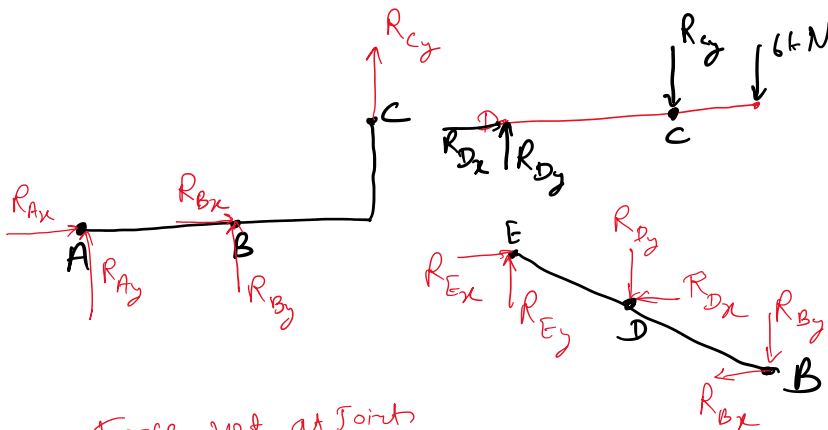
$$\sum F_x = 0$$

$$R_{Ax} + R_{Ex} = 0$$

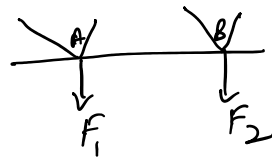
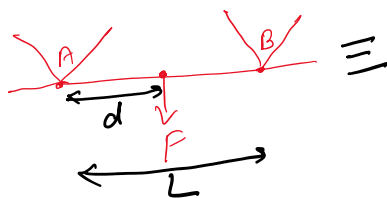
$$R_{Ex} = -15\text{kN}$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Ey} - 6\text{kN} = 0$$



Force w/d at joints



$$\sum F_{II} = \sum F_I$$

$$F_1 + F_2 = F$$

$$(\sum M_A)_{II} = (\sum M_A)_I$$

$$F_2 \times L = Fd$$

$$F_2 = \frac{Fd}{L}$$

$$F_1 = F \left(1 - \frac{d}{L}\right)$$

Static Determinacy

m = number of members

r = number of support reactions

m = number of members

r = number of support reactions

j = number of joints

number of equations = $2j$ for 2D Truss
 $3j$ for 3D Truss

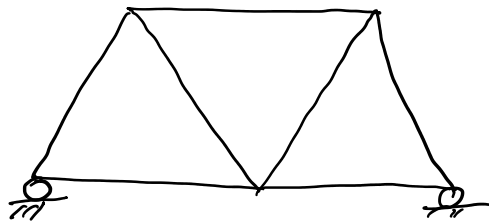
Number of unknowns = $m+r$

$$m+r = \begin{cases} 2j & \text{in 2D} \\ 3j & \text{in 3D} \end{cases}$$

if $m+r > 2j$ ← Indeterminate

if $m+r < 2j$

if $m+r = 2j$



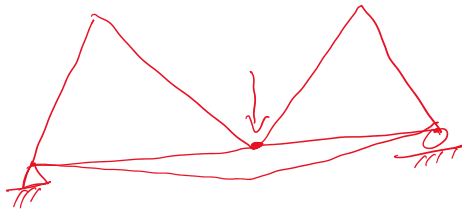
$$m = 7$$

$$j = 5 \Rightarrow 2j = 10$$

$$r = 2$$

$$m+r < 2j$$

Partial Fixity



$$r = 3$$

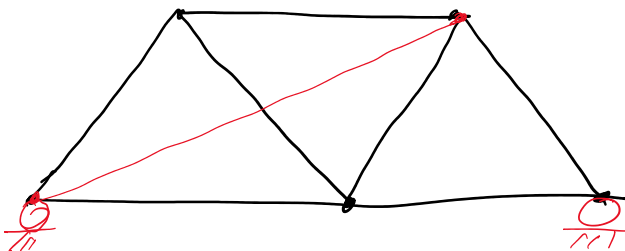
$$j = 5 \Rightarrow 2j = 10$$

$$m = 6$$

$$m+r < 2j$$

Not rigid

(Mechanism)

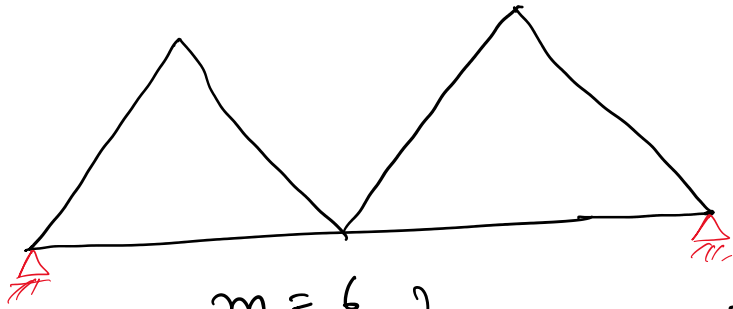


$$m = 8$$

$$r = 2$$

$$j = 5 \Rightarrow m+r = 2j$$

Partial Fixity



$$\begin{aligned}
 m &= 6 \\
 r &= 4 \\
 j &= 5
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow m+r = 2j$$

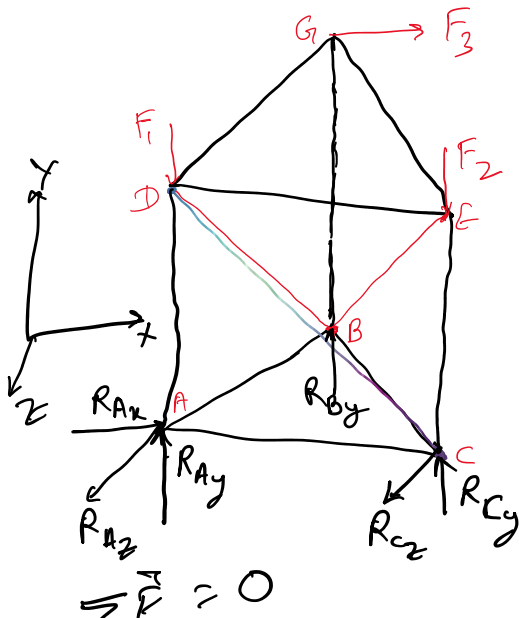
(Mechanism)

3-D Truss

- 3-D structure consisting of 2-force member
- All members are connected to one another by frictionless ball-and-socket joints
- Forces are applied only at joints
- Weightless member



Example



$$\begin{aligned}
 \sum M_A &= 0 \\
 \vec{r}_{B/A} \times \vec{R}_B + \vec{r}_{C/A} \times \vec{R}_C \\
 + \vec{r}_{E/A} \times \vec{F}_E \\
 + \vec{r}_{G/A} \times \vec{F}_G &= 0
 \end{aligned}$$

$$\begin{aligned}
 R_{Cz} &= \\
 R_{Cy} &= \\
 R_{Cx} &=
 \end{aligned}$$

$$\sum \vec{F} = 0$$

$$R_{Cz} \quad \text{and}$$

$$R_{Cy} =$$

$$R_{By} =$$

Method of Section

Cut the structure into two parts such that it

a. cuts the bar of interest and

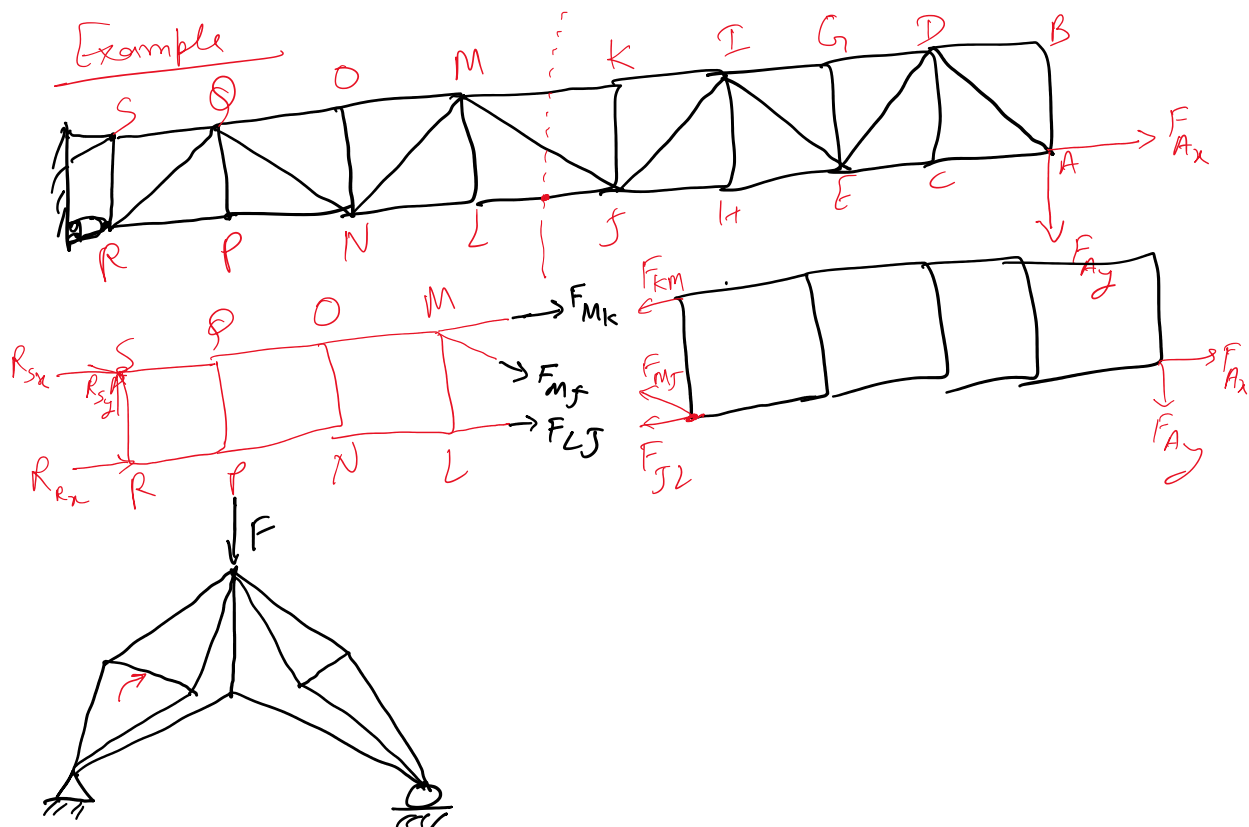
b. cuts at most 3 bars and

c. One of the two parts must have all loads known

i. loads are the externally applied load

ii. reaction forces that you calculate using the FBD of the structure

d. solve set of equilibrium.



Frame



A structure with one or more 3-force or multi-force members

rigid body

multi-force
Rigid body

