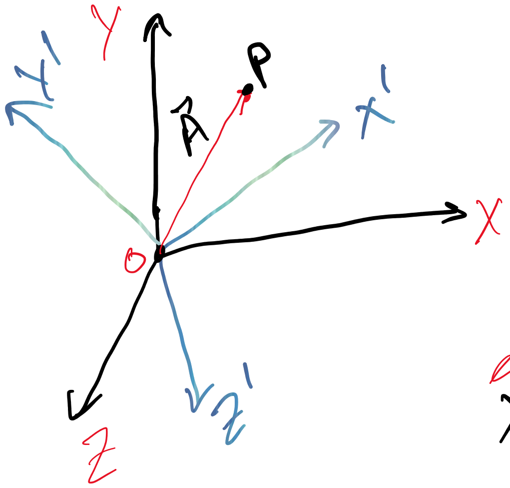
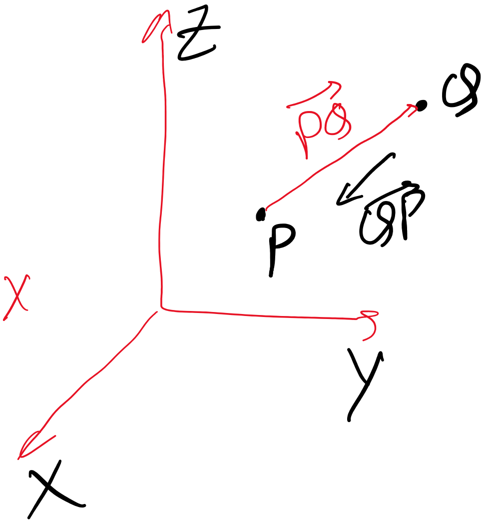
Vectors

Monday, January 6, 2025 9:51 AM



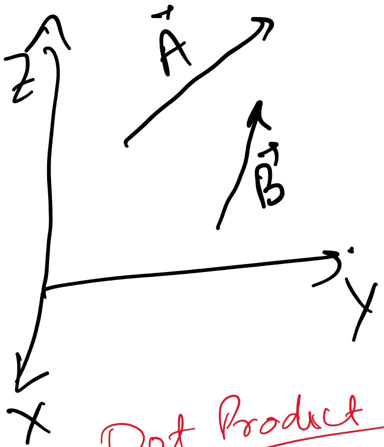


$$\vec{r}_{P/O} = \vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{PQ} = \vec{r}_{Q/P} = -\vec{r}_{P/Q}$$

$$\vec{A} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}$$

$$\left.\begin{aligned}\vec{A} &= A_{x'}\vec{i}' + A_{y'}\vec{j}' + A_{z'}\vec{k}' \\ &= A_x\vec{i} + A_y\vec{j} + A_z\vec{k}\end{aligned}\right\}$$



$$\begin{aligned}\vec{C} &= \vec{A} + \vec{B} \\ &= (A_x + B_x)\vec{i} \\ &+ (A_y + B_y)\vec{j} \\ &+ (A_z + B_z)\vec{k}\end{aligned}$$

Dot Product

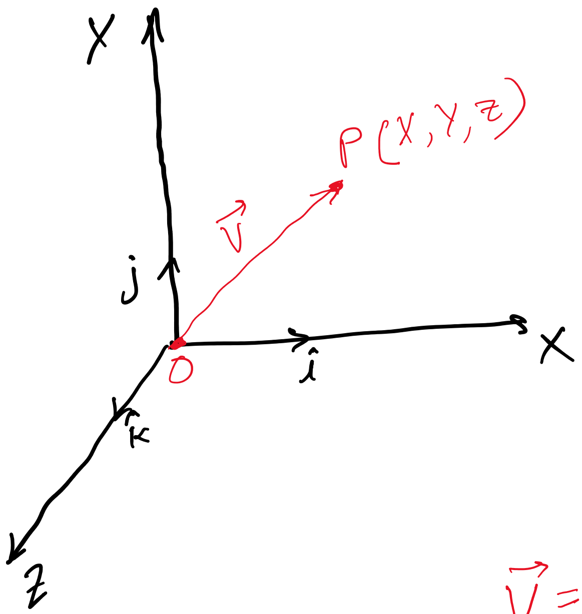
← Scalar Products

x Dot Product

$$C = \vec{A} \cdot \vec{B}$$ ← Scalar Products

Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$



$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k} = \begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix}$$

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{\vec{V} \cdot \vec{V}}$$

$$\vec{V}_1 = V_{1x} \hat{i} + V_{1y} \hat{j} + V_{1z} \hat{k}$$

$$\vec{V}_2 = V_{2x} \hat{i} + V_{2y} \hat{j} + V_{2z} \hat{k}$$

$$\vec{V}_1 \cdot \vec{V}_2 = |\vec{V}_1| \, |\vec{V}_2| \cos\theta$$

θ = angle between $\vec{V}_1$ & $\vec{V}_2$

$$\vec{V} = (\vec{V} \cdot \hat{i})\, \hat{i} + (\vec{V} \cdot \hat{j})\, \hat{j} + (\vec{V} \cdot \hat{k})\, \hat{k}$$

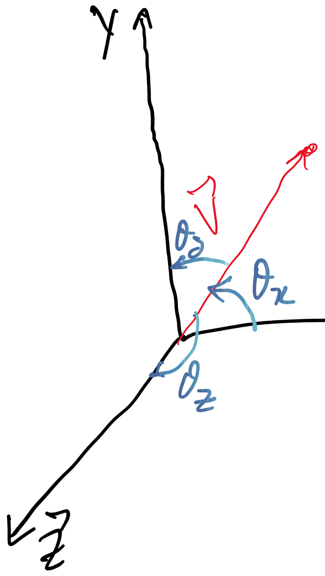
$$\hat{n} = n_x \hat{i} + n_y \hat{j} + n_z \hat{k}$$

$$\vec{V}_n = (\vec{V} \cdot \hat{n})\, \hat{n}$$

$$\vec{V}_{12} = \underbrace{\left[\vec{V}_1 \cdot \left(\frac{\vec{V}_2}{|\vec{V}_2|}\right)\right]}_{\text{magnitude}} \underbrace{\left(\frac{\vec{V}_2}{|\vec{V}_2|}\right)}_{\text{Direction}}$$

Y

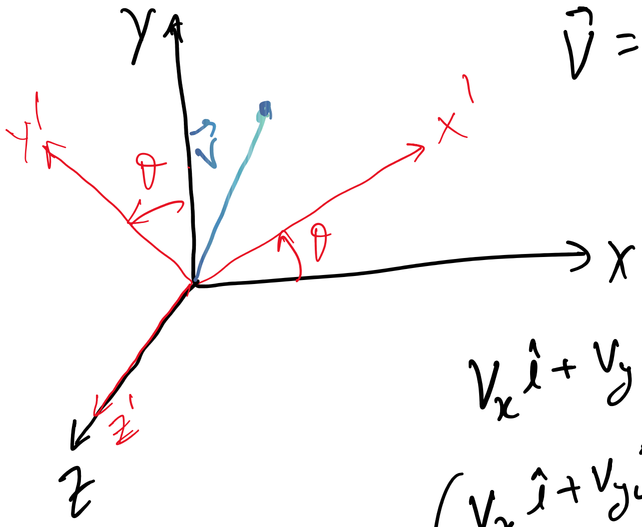
$$\vec{\ } = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$



$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$
$$= (\vec{V}\cdot\hat{i})\,\hat{i} + (\vec{V}\cdot\hat{j})\,\hat{j} + (\vec{V}\cdot\hat{k})\,\hat{k}$$
$$= V\cos\theta_x\,\hat{i} + V\cos\theta_y\,\hat{j} + V\cos\theta_z\,\hat{k}$$

$$\underbrace{\underset{l}{\cos\theta_x}\;,\;\underset{m}{\cos\theta_y}\;,\;\underset{n}{\cos\theta_z}}_{\text{Direction Cosines}}$$

$$\begin{cases} V_x = Vl \\ V_y = Vm \\ V_z = Vn \end{cases} \qquad l^2 + m^2 + n^2 = 1$$



$$\vec{V} = V_x \hat{i} + V_y \hat{j}$$
$$= V_{x'} \hat{i}' + V_{y'} \hat{j}'$$

$$V_x \hat{i} + V_y \hat{j} = V_{x'} \hat{i}' + V_{y'} \hat{j}'$$
$$(V_x \hat{i} + V_y \hat{j})\cdot\hat{i}' = (V_{x'} \hat{i}' + V_{y'} \hat{j}')\cdot\hat{i}'$$
$$V_x (\hat{i}\cdot\hat{i}') + V_y (\hat{j}\cdot\hat{i}') = V_{x'} \underbrace{(\hat{i}'\cdot\hat{i}')}_{1} + V_{y'} \underbrace{(\hat{j}'\cdot\hat{i}')}_{0}$$

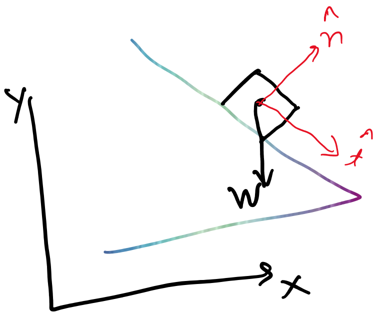
$$V_{x'} = V_x \cos\theta + V_y \sin\theta$$
$$V_{y'} = -V_x \sin\theta + V_y \cos\theta$$

$$\vec{V}' = \begin{Bmatrix} V_{x'} \\ V_{y'} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} V_x \\ V_y \end{Bmatrix}$$

$$\boxed{\vec{V}' = R\,\vec{V}}$$

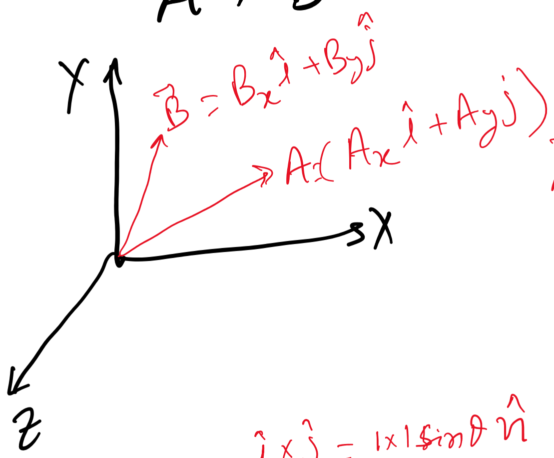
$$\boxed{\vec{V}' = R\vec{V}}$$

$$\vec{F}_w = -w\hat{j}$$



Cross-Product

$$\vec{A} \times \vec{B} = A B \sin\theta_{AB}\, \hat{n}_{AB}$$



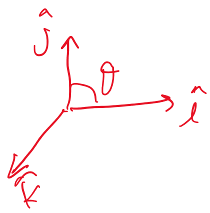
$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j}) \times (B_x\hat{i} + B_y\hat{j})$$

$$= A_xB_x(\hat{i}\times\hat{i}) + A_xB_y(\hat{i}\times\hat{j}) + A_yB_x(\hat{j}\times\hat{i}) + A_yB_y(\hat{j}\times\hat{j})$$

$$= \vec{0} + A_xB_y\hat{k} + A_yB_x(-\hat{k}) + \vec{0}$$

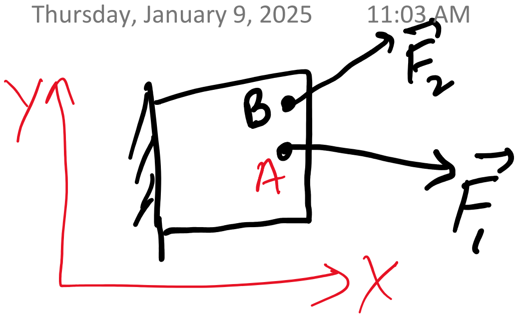
$$\vec{A}\times\vec{B} = (A_xB_y - A_yB_x)\hat{k}$$

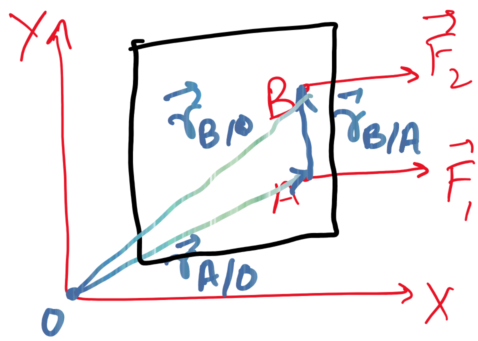
$$\hat{i}\times\hat{j} = 1\times1\sin\theta\,\hat{n}$$



Force System

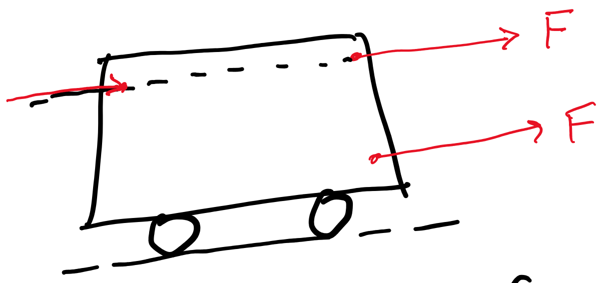
Thursday, January 9, 2025 11:03 AM



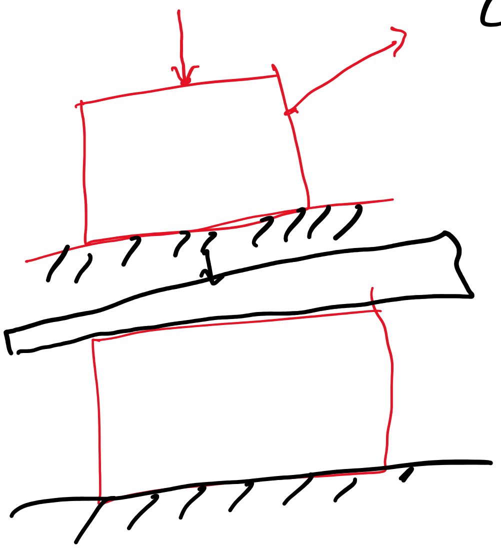


$$\vec{r}_{B/A} = \text{Const.}$$

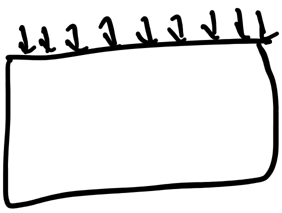
Principle of Transmissibility



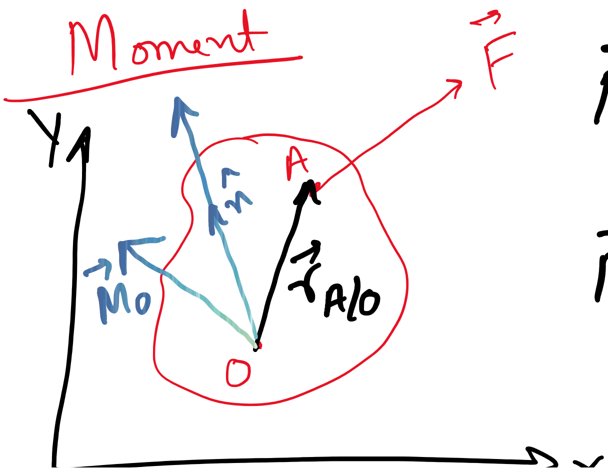
Concentrated Forces



Distributed forces



Moment



$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F}$$

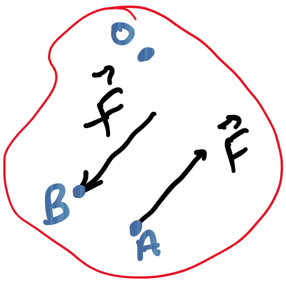
$$\vec{M}_O^n = (\vec{M}_O \cdot \hat{n})\,\hat{n}$$

$$= \left[(\vec{r}_{A/O} \times \vec{F}) \cdot \hat{n}\right]\hat{n}$$

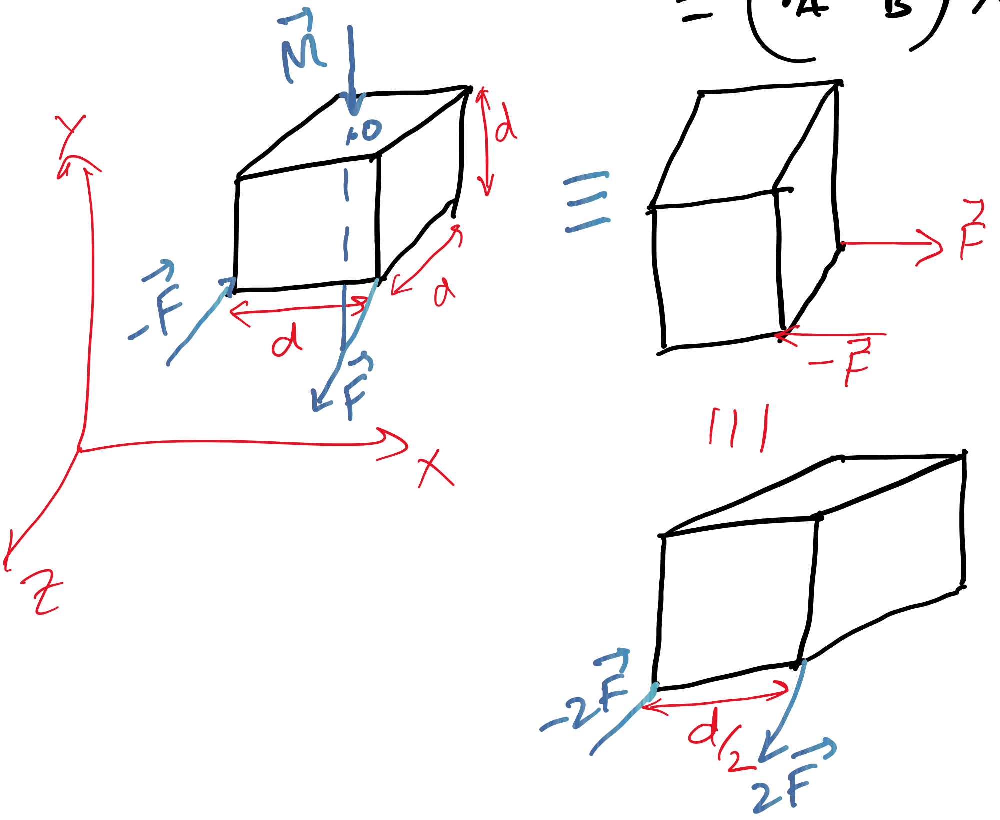
$-[(\vec{r}_{A/O})$

Verignon Theorem

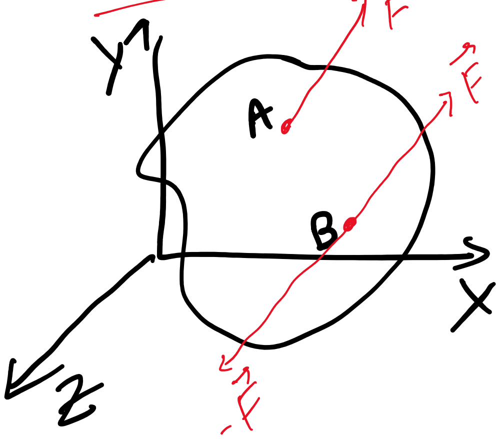
$$\vec{M}_o = \sum_i \vec{r}_i \times \vec{F}_i$$

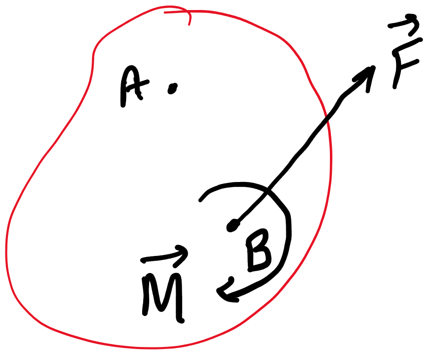


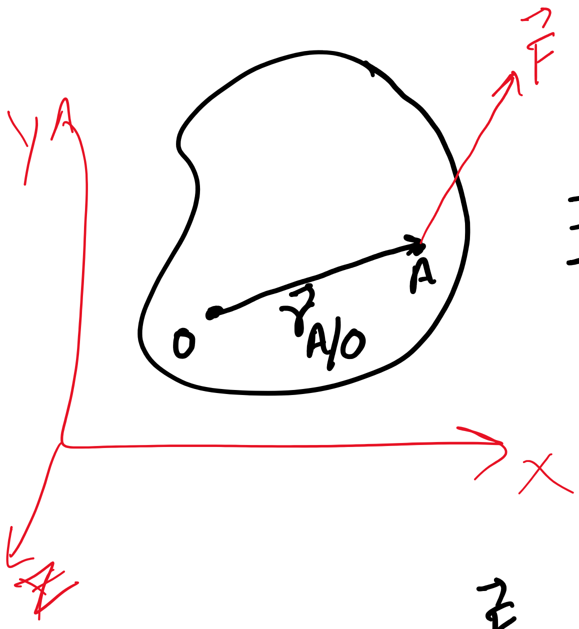
$$\vec{M}_o = (\vec{r}_B \times -\vec{F}) + (\vec{r}_A \times \vec{F})$$
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$



Force System







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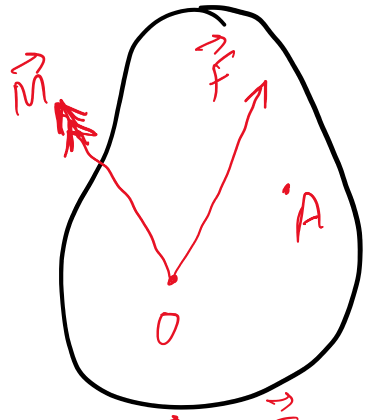
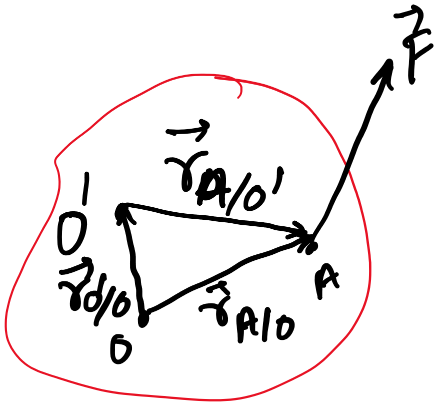
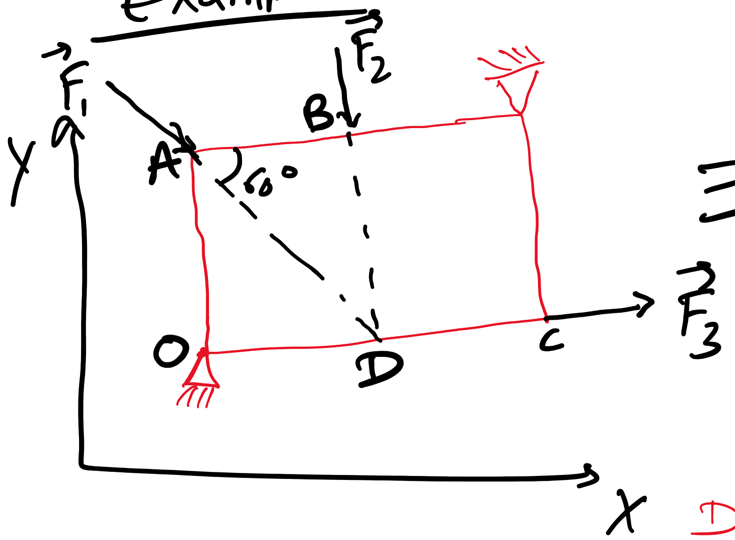


$$\vec{M} = \vec{r}_{A/O} \times \vec{F}$$



$$\vec{M}_{O'} = \vec{r}_{A/O'} \times \vec{F}$$
$$= \left(\vec{r}_{A/O} - \vec{r}_{O'/O}\right) \times \vec{F}$$

Example



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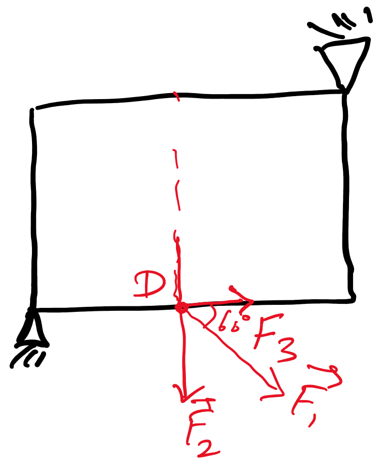
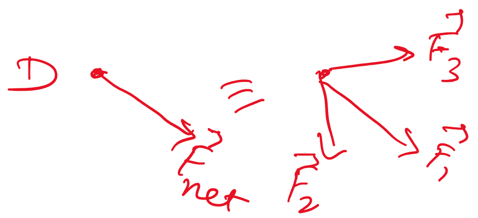
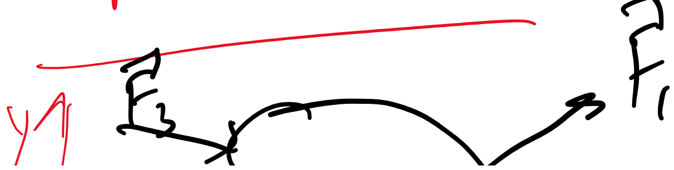
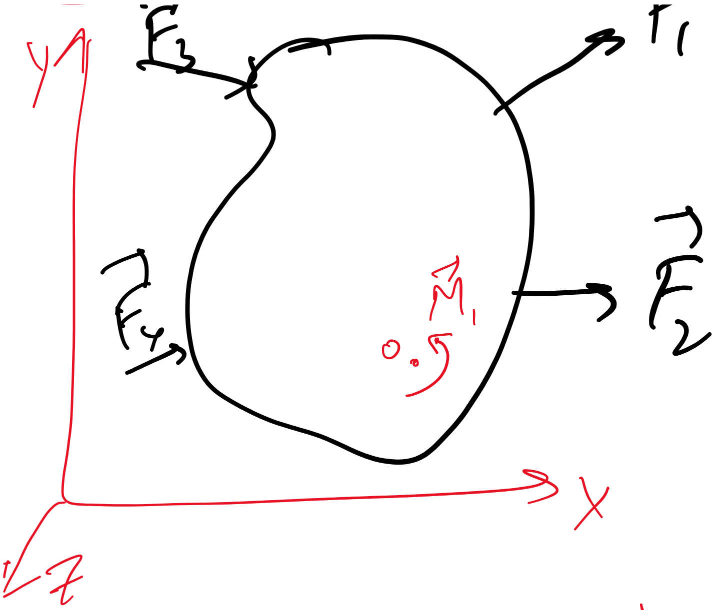




Resultant



$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i$$

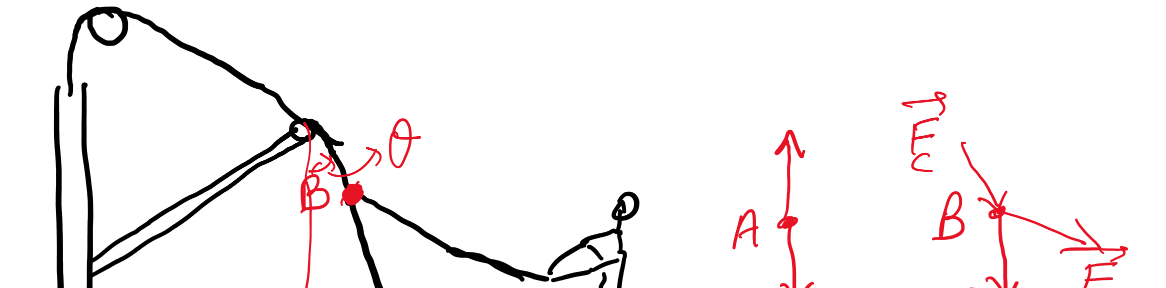


$$F_R = \sum_{i=1} F_i$$

$$\vec{M}_R = \sum M_{i_o}$$

Particle Equilibrium

$$\vec{F}_R = \sum \vec{F}_i = 0$$



$$\sum \vec{F}_A = 0$$

$$\sum \vec{F}_B = 0$$

$$\sum \vec{F} = 0$$

$$\sum F_{x_A} \hat{i} + \sum F_{y_A} \hat{j} + \sum F_{z_A} \hat{k} = 0$$

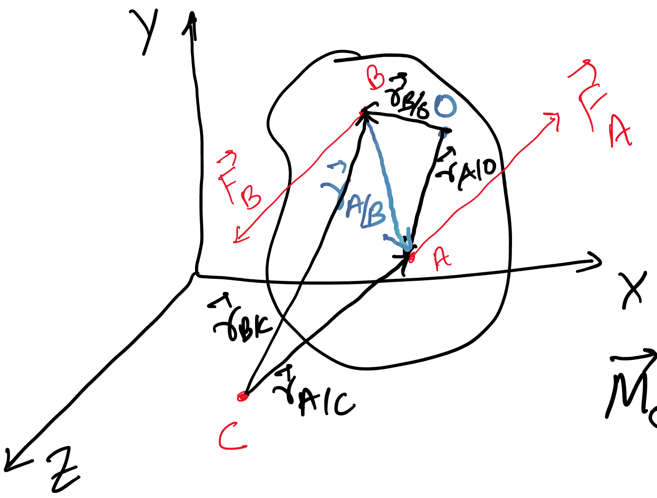
$$\sum F_{x_A} = 0$$

$$\sum F_{y_A} = 0$$

$$\sum F_{z_A} = 0$$

Couple

Wednesday, January 15, 2025 9:05 AM



$$\Sigma \vec{F} = 0$$

$$\vec{F}_A + \vec{F}_B = 0$$

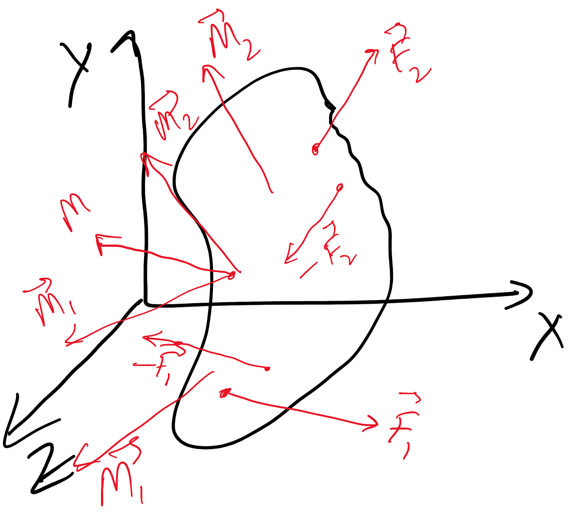
$$\vec{F}_A = -\vec{F}_B = \vec{F}$$

$$|\vec{F}_A| = |\vec{F}_B| = F$$

$$\begin{aligned}
\vec{M}_O &= \vec{r}_{A/O} \times \vec{F}_A + \vec{r}_{B/O} \times \vec{F}_B \\
&= \vec{r}_{A/O} \times \vec{F} + \vec{r}_{B/O} \times (-\vec{F}) \\
&= (\vec{r}_{A/O} - \vec{r}_{B/O}) \times \vec{F} \\
&= \vec{r}_{A/B} \times \vec{F}
\end{aligned}$$

$$\begin{aligned}
\vec{M}_C &= \vec{r}_{A/C} \times \vec{F}_A + \vec{r}_{B/C} \times \vec{F}_B \\
&= \vec{r}_{A/C} \times \vec{F} + \vec{r}_{B/C} \times (-\vec{F}) \\
&= (\vec{r}_{A/C} - \vec{r}_{B/C}) \times \vec{F} \\
&= \vec{r}_{A/B} \times \vec{F}
\end{aligned}$$

$$|\vec{M}_C| = |\vec{M}_O|$$



$$\Sigma \vec{F} = 0$$

$$\vec{F}_1 - \vec{F}_1 + \vec{F}_2 - \vec{F}_2 = 0$$

$$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1$$

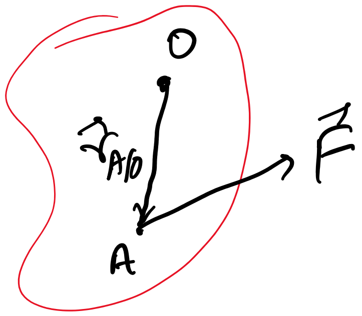
$$\vec{M}_2 = \vec{r}_2 \times \vec{F}_2$$

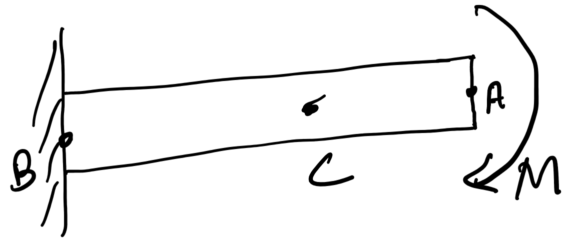
$$\vec{M} = \vec{M}_1 + \vec{M}_2$$

$$\vec{M} = \sum \vec{r}_i \times \vec{F}_i \qquad \vec{M} = \vec{M}_1 + \vec{M}_2$$

$$= \sum \vec{M}_i$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F}$$





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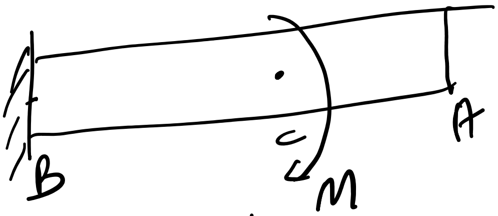

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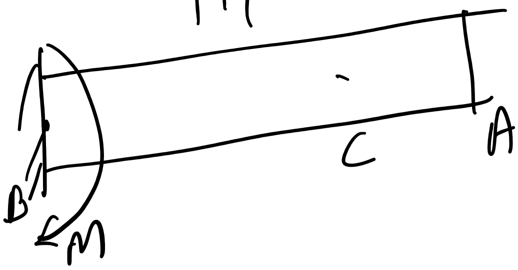
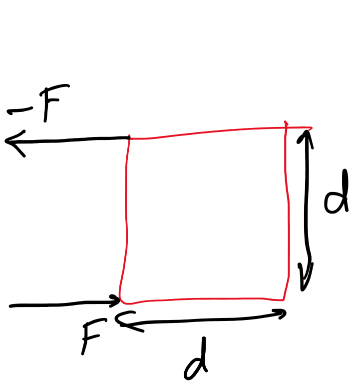
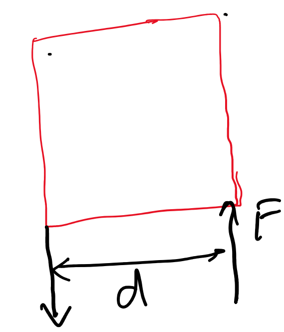
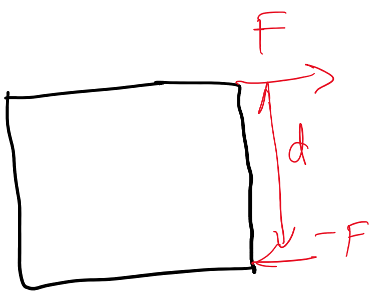




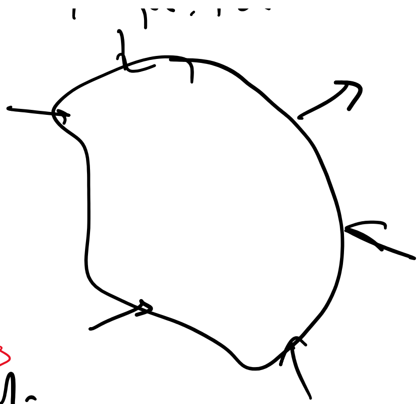
M = Fd



-F M = Fd

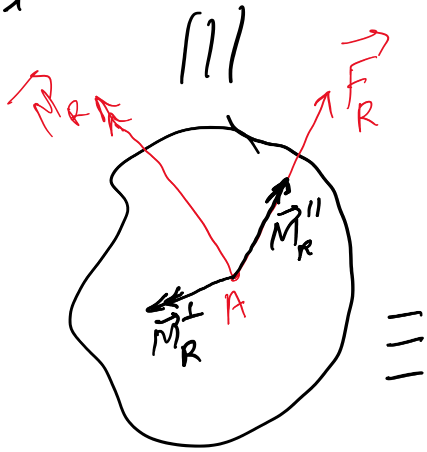


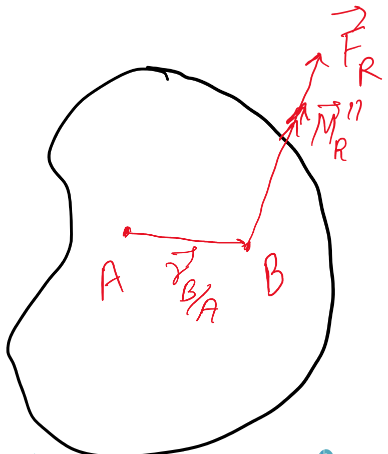
M = Fd



$$\vec{F}_R = \sum \vec{F}_i$$

$$\vec{M}_R = \sum \vec{M}_i$$





Wrench System